

Linear Optics C-Phase gate made simple

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Linear optics quantum logic gates are the best tool to generate multi-photon entanglement. Simplifying a recent approach [1] we were able to implement the conditional phase gate with only one second order interference at a polarization dependent beam splitter, thereby significantly increasing its stability. The improved quality of the gate is evaluated by analysing its entangling capability and by performing full process tomography. The achieved results ensure that this device is well suited for implementation in various multi photon quantum information protocols.

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The quantum computer is one of the most promising and desirable goals in quantum information science. Its implementation relies strongly on the capability to engineer entanglement in the quantum system of choice. For qubits it was shown that entangling gates (like the C-phase gate or the CNOT) together with single qubit operations are sufficient to create any kind of quantum network.

Photons are well suited for quantum information tasks, as their interaction with the environment is small guaranteeing low decoherence. While the *creation* of entangled photon pairs via spontaneous parametric down conversion became a standard technique, its *control* is still a major challenge, mainly due to low nonlinear interaction efficiencies. One solution to this problem is using linear optics components and introducing the nonlinearity via ancillary single photons and photon number resolving detectors [2]. Initial demonstrations showed that such gates can be implemented, once the necessary sources and detectors become available on a larger scale [3]. Another solution, requiring much less resources becomes possible if one focuses on performing only a limited number of quantum logic operations. Then one can control the action of the gate by conditioning it to the detection of one photon in each of the output ports. This will occur only with a certain probability, which, however, might be larger than the one achievable with the first method and equivalent resources. In particular, a controlled phase (C-Phase) gate was introduced recently [1], which uses a combination of first and second order interference to obtain C-Phase action in 1/9 of the cases. Yet, since first-order interference requires stability of the setup on the order of less than the photon's wavelength, for multi-photon experiments [4] more simple and stable implementations surely are desirable.

Here we introduce a linear optics C-Phase gate, which uses only a single two-photon interference at a polarization dependent beam splitter. The stability requirements are thereby relaxed to the coherence length of the detected photons ($\approx 150\mu m$) and can easily be fulfilled without additional stabilization equipment. To characterize the C-phase gate we first study the entangling ca-

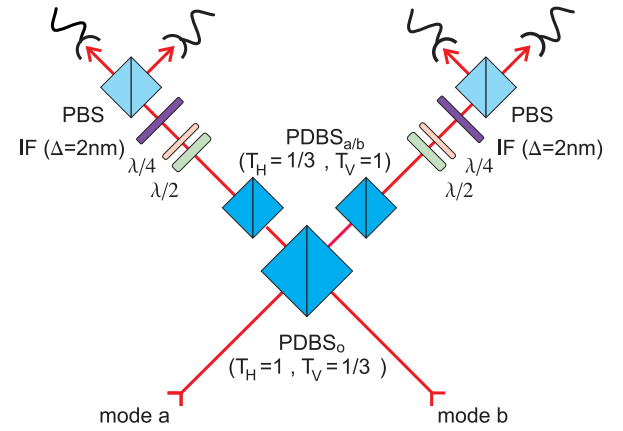


FIG. 1: Experimental setup for the C-Phase gate. The phase is introduced by a second order interference at a polarization dependent beam splitter PDBS_o. To obtain equal output amplitudes for any input polarization state polarization dependent beam splitters with reversed splitting ratio PDBS_{a/b} are placed in each mode. The gate operation is applied in case of a coincidence detection between modes a and b. The resulting output state is analyzed via half- and quarter-wave plates $\lambda/2$, $\lambda/4$ and a polarizing beam splitter PBS.

pability of the gate by determining the fidelity and negativity of the output for four different input states. Second, we use linear quantum process tomography (QPT) [5, 6] to analyze the gate operation. As imperfect interference reduces the quality of the gate and induces state-selective incoherence we had to account for the non-trace-preserving character of the gate. Instead of the usual maximum likelihood approach, we use prior knowledge of the intrinsic features of our setup, in order to obtain physical and easily understandable parameters for characterizing the gate and estimating its performance.

The ideal C-Phase gate acts on two-qubit input states

$$|\psi_{in}\rangle = (c_{HH}|HH\rangle + c_{HV}|HV\rangle + c_{VH}|VH\rangle + c_{VV}|VV\rangle), \quad (1)$$

and applies a relative π -phaseshift to the contribution VV only, such that

$$|\psi_{out}\rangle = (c_{HH}|HH\rangle + c_{HV}|HV\rangle + c_{VH}|VH\rangle - c_{VV}|VV\rangle). \quad (2)$$

Here we encode the logical 0 (1) in linear horizontal H (vertical V) polarization of single photons. c_{HH} denotes the amplitude of the $|HH\rangle$ -term (for the other terms accordingly).

The application of the phase shift relies on second-order interference of indistinguishable photons at a polarization dependent beam splitter (PDBS) (Fig.1) [7, 8]. Two input modes a and b are overlapped at PDBS_O. The transmission of 1/3 for vertical polarization results in a total amplitude of $-1/3$ for the $|VV\rangle$ output terms, as can be seen by adding the amplitudes for a coincident detection:

$$(t_V^a \cdot t_V^b) + (ir_V^a \cdot ir_V^b) = \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}} - \sqrt{\frac{2}{3}}\sqrt{\frac{2}{3}} = -1/3 \quad (3)$$

where t_i^x (r_i^x) is the amplitude for transmission (reflection) of state $|i\rangle$ in mode x . Perfect transmission of horizontal polarization causes that no interference happens on the contributions $|HH\rangle$, $|HV\rangle$ and $|VH\rangle$. As the absolute values of the amplitudes need to be equal for any input we still need to attenuate the contributions that include horizontal polarization. This is achieved by PDBS_{a/b} with the transmission 1/3 for horizontal polarization and perfect transmission for vertical polarization in both output modes. All together we find a probability of 1/9 to obtain a coincidence in the outputs and thus a gate operation with perfect fidelity.

Working with real components results in deviations from the theoretical derivation. A detailed calculation with arbitrary transmission and reflection amplitudes at PDBS_O and PDBS_{a,b} shows how their parameters influence the gate operation. In general we obtain from $|\psi_{in}\rangle$

$$\begin{aligned} |\psi_{out}\rangle = & (c_{HH}t_H^at_H^bt_H^a b_H - c_{HH}r_H^ar_H^br_H^a b_H)|HH\rangle \\ & + (c_{HV}t_H^at_V^b a_H b_V - c_{HV}r_H^ar_V^b a_H b_V)|HV\rangle \\ & + (c_{VH}t_V^at_H^b a_V b_H - c_{VH}r_V^ar_H^b a_V b_H)|VH\rangle \\ & + (c_{VV}t_V^at_V^b a_V b_V - c_{VV}r_V^ar_V^b a_V b_V)|VV\rangle, \end{aligned} \quad (4)$$

where a_i (b_i) are the transmission amplitudes of $|i\rangle$ in mode a (b). To obtain the expected C-phase gate operation one has to fulfill several conditions, which give an insight in how the setup has to be built. First, $(r_V^ar_V^b)/(t_V^at_V^b) = 2$, which is approximately achieved by slightly varying the angle of incidence at PDBS_O. Experimentally we reach a value of 2.018 ± 0.003 . Second,

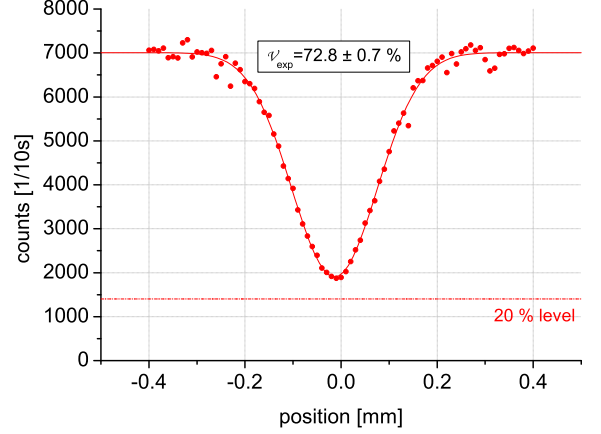


FIG. 2: HOM-Interference at the polarization dependent overlap beam splitter in the phase gate for a $|VV\rangle$ input. In case of perfect interference the countrate should drop down to 20 % leading to a theoretically achievable dip visibility of 80 %.

$r_H^a = 0 = r_H^b$, which requires the reflection of the horizontal polarization at the overlap beam splitter to be zero. The third condition, $t_H^a a_H = t_V^a a_V$, and $t_H^b b_H = t_V^b b_V$, respectively, determines the setting for the attenuation at PDBS_{a,b}.

To experimentally test the gate operation we used photon pairs emitted from spontaneous parametric down conversion. A 2 mm thick BBO (β -Barium Borate) crystal was pumped by UV pump pulses with a central wavelength of 390 nm and an average power of 700 mW from a frequency-doubled mode-locked Ti:sapphire laser (pulse length 130 fs). The pulsed operation is not necessary when working only with photon pairs, but as the gate is intended to work in multi-photon applications we preferred to characterize it for this mode of operation. The emission is filtered with polarizers to prepare input product states with high quality. We couple the photon pairs into single mode fibers for selection of the spatial modes. This guarantees identical beam modes which eases the alignment of spatial mode matching at PDBS_O. The spectral mode selection is improved via 2nm bandwidth filters behind the gate.

To ensure the same optical path length between the crystal and the overlap beam splitter for both photons, one of the output couplers of the single mode fibers is mounted on a translation stage. The position of zero delay is determined from the minimum of the coincidence rate for $|VV\rangle$ -input (Hong-Ou-Mandel [7], "HOM", Dip Fig. 2). In each output mode of the C-Phase gate the polarization is analyzed via quarter and half waveplates and a polarizing beam splitter with single photon avalanche photo diodes. For the analysis of the final two-photon states the coincidence count rates for each of the four contributions have to be corrected for the different de-

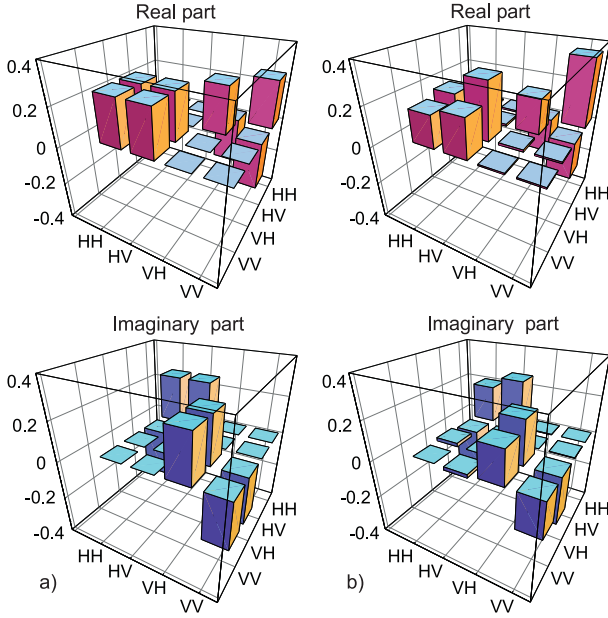


FIG. 3: (a) Theoretically expected and (b) experimentally obtained gate output density matrix for $|L+\rangle$ input state.

tector efficiencies. The errors on all quantities are deduced from propagated Poissonian counting statistics of the raw detection events and efficiencies.

The HOM-measurement shown in Fig. 2 also gives information about the indistinguishability of the photons at the PDBS. For large delay, the two photons are completely distinguishable due to their time of arrival. The probability to get a coincidence from a $|VV\rangle$ -input is then $5/9$. In case of perfectly indistinguishable photons at zero delay, the probability drops to $1/9$. The Dip-Visibility is defined by $\mathcal{V} = (c_\infty - c_0)/c_\infty$, where c_0 is the count rate at zero delay and c_∞ at positions with big delay. From the above considerations we obtain a theoretical value of $\mathcal{V}_{th} = 80\%$, and experimentally, applying least-square fit, we find $\mathcal{V}_{exp} = 72.8\% \pm 0.7\%$. We call $\mathcal{Q} = \mathcal{V}_{exp}/\mathcal{V}_{th} = 91.0\% \pm 0.9\%$ the overlap quality. One can conclude that the amount of additional $|VV\rangle\langle VV|$ -noise depends on the input, but is 9% at maximum.

As a first step in the analysis of the performance of our gate we look at its capability to entangle. We choose $|++\rangle, |L+\rangle, |L+\rangle$, and $|LL\rangle$, with $|+\rangle = 1/\sqrt{2}(|H\rangle + |V\rangle)$ and $|L\rangle = 1/\sqrt{2}(|H\rangle + i|V\rangle)$, as input product states and perform state tomography on the output states [9]. We use linear tomography as the resulting matrices all have eigenvalues greater or equal -0.02, i.e., are almost physical without corrections. For an ideal C-phase gate one would obtain a maximally entangled output for these input states, for example

$$\begin{aligned} |L+\rangle &= 1/2(|HH\rangle + i|VH\rangle + |HV\rangle + i|VV\rangle) \\ \overline{PG}|L+\rangle &= 1/2(|HH\rangle + i|VH\rangle + |HV\rangle - i|VV\rangle) \\ &= 1/\sqrt{2}(|LH\rangle + |RV\rangle). \end{aligned} \quad (5)$$

The experimentally observed fidelities relative to the ex-

pected output states are all better $F_{exp} \geq 80.5\% \pm 0.6\%$. Fig. 3 exemplarily shows the experimental result for $|L+\rangle$ -input ($F_{exp}^{L+} = 87.8\% \pm 0.6\%$). Note that for states with a fidelity larger than $(2 + 3\sqrt{2})/8 = 0.78$ CHSH-inequalities are violated ([10]), which is the case for all of our examples. To quantify the entanglement we also calculated the logarithmic negativity — for all output states we find $\mathcal{N}_{exp} \geq 0.73 \pm 0.02$ ($\mathcal{N}_{exp}^{L+} = 0.75 \pm 0.02$).

For a complete characterization of an arbitrary unknown process one can use quantum process tomography (QPT). For QPT the process is represented by a super-operator \mathcal{E} which is decomposed in a linear combination of a basis of unitary transformations E_i :

$$\mathcal{E}(\rho_{in}) = \sum_{i,j} \chi_{ij} E_i \rho_{in} E_j^\dagger \quad (6)$$

The matrix χ completely describes the process. In order to obtain all components χ_{ij} , the normalized output density matrices ρ_{out}^k for a tomographic set of, usually separable, input states are measured, in our case for the inputs ($|HH\rangle, |HV\rangle, |H+\rangle, |H-\rangle, |VH\rangle, |VV\rangle, |V+\rangle, |VL\rangle, |LH\rangle, |L+\rangle, |L-\rangle, |LL\rangle$). As the contribution of the $|VV\rangle\langle VV|$ -noise is input state dependent our process is non-trace-preserving. This means that the outcomes occur with different probabilities p_k [5] for the different input states $\rho_{in}^k \text{Tr}(\mathcal{E}(\rho_{in}^k))$:

$$\begin{aligned} \rho_{out}^k &= \frac{\mathcal{E}(\rho_{in}^k)}{\text{Tr}(\mathcal{E}(\rho_{in}^k))} \\ \Rightarrow \mathcal{E}(\rho_{in}^k) &= \text{Tr}(\mathcal{E}(\rho_{in}^k)) \rho_{out}^k = p_k \rho_{out}^k. \end{aligned} \quad (7)$$

We determine these probabilities from the diagonal entries of all measured output density matrices. The normalized density matrices together with the corresponding probabilities can be used to evaluate χ_{ij} via Eq. (7) and (6). To account for the probabilistic nature of the gate an overall normalization is performed such that $p_{HH} = 1/9$.

Fig. 4a shows the process matrix χ_{th} of the ideal linear optics phase gate. It represents the decomposition of the C-Phase gate into unitary operations, for our choice of E_i resulting in

$$\overline{PG}_{ideal} = (\mathbf{1} \otimes \mathbf{1} + \sigma_z \otimes \mathbf{1} + \mathbf{1} \otimes \sigma_z - \sigma_z \otimes \sigma_z)/3. \quad (8)$$

The four peaks in the diagonal of χ_{th} show the equal weights of the contributions, while the negative entries at the edges represent the negative sign at $\sigma_z \otimes \sigma_z$. This matrix now can be compared with the experimentally obtained one (Fig. 4b). Only the real parts are shown since the imaginary parts are close to zero (average 0.0 ± 0.002). As the introduced noise is not too big, the experimentally measured process matrix still demonstrates nicely the features of the gate operation. The main differences are the lower non-diagonal terms indicating reduced coherence in the system. From the estimated process tomography matrix we calculated a process fidelity of $F_p = \text{Tr}(\chi_{th} \cdot \chi_{exp}) / (\text{Tr}(\chi_{th}) \cdot \text{Tr}(\chi_{exp})) = 81.8\%$.

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